Positive solutions of fully nonlinear degenerate Lane-Emden type equations

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Classical and new methods in

Calculus of Variations and PDEs

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Truncated Laplacians

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Some examples

- $F(X) = \sum_{i=1}^k \lambda_{j_i}(X)$, $1 \leq j_1 \leq \ldots \leq j_k \leq N$ $\qquad k \leq N$
- $F(X) = \sum_{i=1}^N \alpha_i \lambda_i(X)$, $\sum \alpha_i = 1$ $\qquad k = 1$
  $\alpha_i = \frac{1}{N} \implies F = \frac{1}{N}\Delta$
  $\alpha_i = 1,\ \alpha_{j\neq i} = 0 \implies F(\cdot) = \lambda_i(\cdot)$
- $F(X) = \max\{\lambda_1(X) + \lambda_4(X), \lambda_2(X) + \lambda_3(X)\}$ $\qquad k = 2$

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- If $\underline{k < N}$ then for any $v \in \mathbb{R}^N$, $|v| = 1$

$$\min_{X \in \mathbb{S}^N} \left(\mathcal{P}_k^{\pm}(X + v \otimes v) - \mathcal{P}_k^{\pm}(X) \right) = 0$$

Strong degeneracy

(using $\operatorname{spec}(v \otimes v) = \{0, \ldots, 0, 1\}$)

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Some references

▷**Differential Geometry** [Sha '87, Wu '87, Ambrosio-Soner '90]

▷**Convex Analysis/PDEs** [Oberman '07, Oberman-Silvestre '11]

▷**PDEs** [Harvey-Lawson '09, Caffarelli-Li-Nirenberg '09, C. Dolcetta-Leoni-Vitolo '16, Cirant-Payne '17, Vitolo '17, Blanc-Rossi '18]

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(2) $\Delta u + u^p = 0$ in $\mathbb{R}^N$ vs $\mathcal{P}_k^{\pm}(D^2u) + u^p = 0$ in $\mathbb{R}^N$

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  Moreover $\forall x_0 \in \partial\Omega$ and $\forall q < \frac{1}{1-p}$

$$\lim_{x \to x_0} \frac{U(x) - U(x_0)}{|x - x_0|^q} = 0 \qquad \textbf{(Anti-Hopf lemma)}$$

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$$u(|x|) = \begin{cases} \left(\frac{1-p}{2k}\left(R^2 - |x-x_0|^2\right)\right)^{\frac{1}{1-p}} & \text{sol. in } B_R(x_0) \subseteq \Omega \\ 0 & \text{in } \Omega \backslash B_R(x_0) \end{cases}$$

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- $p < 1$ To get uniqueness we restric to

$$\begin{cases} \mathcal{P}_k^-(D^2u) + u^p = 0 & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u = 0 & \text{su } \partial\Omega \end{cases}$$

$u > 0 + p < 1 \Rightarrow$ Comparison principle
Existence based on Perron method: upper and lower barriers (here the convexity of Ω is exploited).

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$$u \leq \left(\frac{R^2}{2k}(1-q)\right)^{\frac{1}{1-q}} \longrightarrow 0 \quad \text{as } q \to 1 \qquad \square$$

Link with Berestycki-Nirenberg-Varadhan

$$\lambda(F,\Omega) := \sup\left\{\mu \in \mathbb{R} \,|\, \exists \phi > 0 \text{ in } \Omega \text{ t.c. } F(D^2\phi) + \mu\phi \le 0\right\}$$

In [Birindelli-G.-Ishii, '18] it has been proved that

- $\mu < \lambda(F,\Omega) \Rightarrow F(D^2\cdot) + \mu\cdot$ satisfies the maximum principle
- $\lambda(\mathcal{P}_k^-,\Omega) = \infty$

The operators

$$\mathcal{P}_k^-(D^2\cdot) + \mu\cdot = \lambda_1(D^2\cdot) + \ldots + \cancel{\lambda_N(D^2\cdot)} + \mu\cdot$$

satisfy the **Maximim principle for any** μ

$F(D^2u) + u^p \ge 0$

If $\lambda(F,\Omega) = \infty$ and $p \ge 1$ then

$$F(D^2u) + \left\|u^{p-1}\right\|_\infty u \ge 0 \implies u \equiv 0$$

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- $p \in (\frac{k}{k-2}, \frac{k+2}{k-2}) \implies \nexists$ **radial positive classical solutions**

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Teorema [G., JDE '19]

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- $p \geq 1$ Topological degree argument $\rightarrow$ A priori estimates $\rightarrow$ Liouville theorems in $\mathbb{R}^N$ o in $\mathbb{R}^N_+$ & **compactness** in $C(\mathbb{R}^N)$
  If $k = 1$ compactness results are consequence of Lipschitz estimates [Brinidelli-G.-Ishii, '18]

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- Open problem: regularity for $1 < k < N$ (too hard for me!)

Thank you for your attention!!!